

S E C O N D E D I T I O N

MASTERING

TECHNICAL

MATHEMATICS

STAN GIBILISCO
NORMAN CROWHURST

Mastering Technical Mathematics

This page intentionally left blank.

Mastering Technical Mathematics

SECOND EDITION

Stan Gibilisco
Norman Crowhurst

McGraw-Hill

New York San Francisco Washington, D.C. Auckland Bogotá
Caracas Lisbon London Madrid Mexico City Milan
Montreal New Delhi San Juan Singapore
Sydney Tokyo Toronto

McGraw-Hill

A Division of The McGraw-Hill Companies



Copyright © 1999 by The McGraw-Hill Companies. All rights reserved. Manufactured in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of the publisher.

0-07-137859-6

The material in this eBook also appears in the print version of this title: 0-07-024828-1.

All trademarks are trademarks of their respective owners. Rather than put a trademark symbol after every occurrence of a trademarked name, we use names in an editorial fashion only, and to the benefit of the trademark owner, with no intention of infringement of the trademark. Where such designations appear in this book, they have been printed with initial caps.

McGraw-Hill eBooks are available at special quantity discounts to use as premiums and sales promotions, or for use in corporate training programs. For more information, please contact George Hoare, Special Sales, at george_hoare@mcgraw-hill.com or (212) 904-4069.

TERMS OF USE

This is a copyrighted work and The McGraw-Hill Companies, Inc. ("McGraw-Hill") and its licensors reserve all rights in and to the work. Use of this work is subject to these terms. Except as permitted under the Copyright Act of 1976 and the right to store and retrieve one copy of the work, you may not decompile, disassemble, reverse engineer, reproduce, modify, create derivative works based upon, transmit, distribute, disseminate, sell, publish or sublicense the work or any part of it without McGraw-Hill's prior consent. You may use the work for your own noncommercial and personal use; any other use of the work is strictly prohibited. Your right to use the work may be terminated if you fail to comply with these terms.

THE WORK IS PROVIDED "AS IS". MCGRAW-HILL AND ITS LICENSORS MAKE NO GUARANTEES OR WARRANTIES AS TO THE ACCURACY, ADEQUACY OR COMPLETENESS OF OR RESULTS TO BE OBTAINED FROM USING THE WORK, INCLUDING ANY INFORMATION THAT CAN BE ACCESSED THROUGH THE WORK VIA HYPERLINK OR OTHERWISE, AND EXPRESSLY DISCLAIM ANY WARRANTY, EXPRESS OR IMPLIED, INCLUDING BUT NOT LIMITED TO IMPLIED WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. McGraw-Hill and its licensors do not warrant or guarantee that the functions contained in the work will meet your requirements or that its operation will be uninterrupted or error free. Neither McGraw-Hill nor its licensors shall be liable to you or anyone else for any inaccuracy, error or omission, regardless of cause, in the work or for any damages resulting therefrom. McGraw-Hill has no responsibility for the content of any information accessed through the work. Under no circumstances shall McGraw-Hill and/or its licensors be liable for any indirect, incidental, special, punitive, consequential or similar damages that result from the use of or inability to use the work, even if any of them has been advised of the possibility of such damages. This limitation of liability shall apply to any claim or cause whatsoever whether such claim or cause arises in contract, tort or otherwise.

DOI: 10.1036/0071378596

**To Samuel, Tim, and Tony
from Uncle Stan**

This page intentionally left blank.

Contents

Introduction *xix*
Acknowledgments *xxi*

Part 1. Arithmetic as an outgrowth of learning to count

1. From counting to addition 3

Counting in tens and dozens 4
Writing numbers greater than ten 5
Why zero is used in counting 6
Man's earliest computer: the abacus 6
By tens and hundreds to thousands 7
Don't forget the zeros 9
Beyond thousands: millions and more 10
Different ways of viewing big numbers 10
Addition is counting on 11
Adding three or more numbers 12
Adding larger numbers 14
Carrying 14
Successive addition 16
The old-time way 16
Checking answers 18
Weights 19
Liquid and dry measures 22
Questions and problems 22

2. Subtraction 24

Subtraction is counting away 24
Checking subtraction by addition 26

Borrowing	26
Subtracting with larger numbers	27
Subtracting cash	28
Making change	29
Subtracting weights	30
Using a balance	31
Subtracting liquid and dry measures	32
<i>Questions and problems</i>	33

3. Multiplication 34

A short cut for repeated addition	34
Use of tables	35
Patterns in numbers	35
How calculators multiply	37
Putting together how people did it	37
Carrying in multiplication	39
A matter of order	40
Using your pocket calculator to verify this process	40
Skipping zeros	41
Either number can be the multiplier	41
Using subtraction in multiplication	42
Multiplying by factors	44
Multiplying with weights	44
Multiplying lengths	45
Multiplying measures	45
<i>Questions and problems</i>	48

4. Division 50

Division began as counting out	50
How a calculator does it	51
Division is multiplication in reverse	52
Dividing into longer numbers	52
Multiplication checks division	53
More about how a calculator does it	54
Dividing by larger numbers (the people way)	54
Multiplication as a “check again”	56
Division by factors	56
Which method is best?	57
When a remainder is left	57
What does the remainder mean?	59
How a calculator handles fractions	61
Fractions that have multiple parts	62
Decimal equivalents of fractions	62
More difficult fractions	63

Where more figures repeat	64
Decimal for one eleventh and others	64
Converting recurring decimals to fractions	65
Where more than one digit recurs	66
<i>Questions and problems</i>	67

5. Fractions 69

Different fractions with the same value	69
Factors help find the simplest form—cancellation	69
Spotting the factors	70
Rules for finding factors	71
How far to try	72
Squares and primes	72
Factoring with a calculator	73
Adding and subtracting fractions	74
Finding the common denominator	75
Calculators that “do” fractions	76
Significant figures	77
Approximate long division: why use it	78
Longhand procedure	79
Using a calculator to find significance	79
Approximate long multiplication	81
<i>Questions and problems</i>	81

6. Area: the second dimension 84

Scales of length: units and measurement	84
Length times length is area	85
What is square?	85
The right angle	86
Different shapes with the same area	87
Square measure	88
From oblongs to triangles	89
Parallelograms	89
Area of parallelograms	90
Area of acute triangles	91
Area of obtuse triangles	92
Area of triangles	93
Metric measure	94
The metric system	95
Area problems	95
<i>Questions and problems</i>	98

7. Time: the fourth dimension 100

- What is dimension? 100
- The fourth dimension: time 101
- Using time to build more dimensions 102
- Average speed 103
- The reference quantity 105
- Changing the average 107
- Making up time 107
- Rate of growth 108
- Fractional increase 109
- Percentages 109
- Percentages with money 111
- Percentages up and down 112
- Graphical representation of facts 113
- Graphs 114
- Questions and problems* 117

**Part 2. Introducing algebra, geometry, and trigonometry
as ways of thinking in mathematics**

8. First notions leading into algebra 123

- Shorter methods for longer problems 123
- Graphs check arithmetic and find solutions 125
- Algebra: a more direct way 125
- Writing it as algebra 126
- Different ways of writing in arithmetic and algebra 127
- Brackets or parentheses 127
- Using more than one set 128
- A problem expressed by algebra 129
- Removing parentheses to solve it 130
- Putting a problem into algebra 130
- Solving it by removing the parentheses 131
- Checking your answer and your work 131
- Magic by algebra 132
- Minus times a minus makes a plus 135
- Solving the problem 135
- Arithmetic numbers in algebra 136
- Number problems 136
- Questions and problems* 137

9. Developing “school” algebra 140

- Orderly writing in algebra 140
- Indices show “place” in algebra 141

Dimension in algebra	142
Expressions, equations, etc.	142
An equation as an action statement	143
Using an equation to solve a problem	144
Simultaneous equations	145
Simultaneous equations solve a fraction problem	146
Solving the problem	147
Solving by substitution	148
Solving for reciprocals	149
Long division clarifies how algebra works	150
Long division finds factors in algebra	151
<i>Questions and problems</i>	152

10. Quadratics 154

Problems with two or more answers	154
Quadratic graph is a symmetrical curve	155
Solving a quadratic equation	156
Using factors to solve equations	157
Finding factors to solve quadratics	158
How factors solve quadratics	158
When factors are even more difficult to find	159
Completing the square	160
Completing the solution by completing the square	161
Checking the answers	162
What the answers mean	163
Comparing methods	164
Formula method	165
Solving by formula	166
Checking results	169
A quadratic problem	170
<i>Questions and problems</i>	171

11. Finding short cuts 173

Difference of squares is always sum times difference	173
Sum and difference in geometry	175
Difference of squares finds factors	175
One way to find a square root	176
“Continued” square root	179
Importance of place in square root	179
Importance of signs in successive roots	180
Imaginary numbers	180
Imaginary numbers find the other two cube roots	181
Simultaneous quadratics	183

Always check! 184
Questions and problems 184

12. Mechanical mathematics 187

Relationship between force and work 187
Measure of force 187
Speed and distance 189
Acceleration and distance 190
Force and work 192
Work and energy 193
Energy and power 194
Gravity as a source of energy because of position 195
Weight as force 196
Gravitational measure of work 197
Energy for constant acceleration 199
Kinetic energy and velocity 199
Acceleration at constant power 200
A stressed spring stores energy 201
Spring transfers energy 202
Resonance cycle 203
Travel and velocity in resonance system 204
Questions and problems 206

13. Ratio in mathematics 209

Proportion or ratio 209
Manipulation of ratio 209
Applying the principle to bigger problems 211
Shape and size 213
About angles in triangles 213
Use of square-cornered triangles 214
Angles identified by ratios 215
Special fact about the right triangle 216
Names for angle ratios 218
Spotting the triangle 219
Degree measure of angles 220
Finding trig ratios for certain angles 221
The right isosceles triangle 222
Other angles 223
Using trigonometry in problems 223
Questions and problems 226

14. Trigonometry and geometry conversions 229

Ratios for sum angles 229
Finding $\sin(A + B)$ 230
Finding $\cos(A + B)$ 231

Finding $\tan(A + B)$	232
Ratios for 75 degrees	233
Ratios of angles greater than 90 degrees	234
Ratios for difference angles	235
Sum and difference formulas	236
Ratios through the four quadrants	237
Pythagoras in trigonometry	238
Multiple angles	239
Properties of the isosceles triangle	240
Angles in a circle	242
Definitions	243
<i>Questions and problems</i>	244

Part 3. Developing algebra, geometry, trigonometry, and calculus

15. Systems of counting 249

Degrees of accuracy	249
Fractions in extended system counting	250
Orders of magnitude	251
Systems of counting	253
Duodecimal system	254
Conversion from decimal to duodecimal	255
Conversion from duodecimal to decimal	256
Binary counting	256
Converting decimal to binary	257
Binary multiplication	258
Alternative binary conversion	260
Binary division	260
Special calculator binary	262
Indices	262
Roots: inverse of powers	264
Surds and indices	265
<i>Questions and problems</i>	265

16. Progressions 268

Arithmetic progression	268
Geometric progression	269
Harmonic progression	270
Sum of an arithmetic series	271
Sum of a geometric series	272
Converging series	274
Sum of a converging series	275

Rate of convergence 276
Permutations 277
Factorial notation! 278
Combinations 279
Powers of a binomial 280
Binomial expansion 281
Binomial series 282
Completing some patterns 283
Using binomial to find roots 284
Making a series converge 285
Questions and problems 286

17. Putting progressions to work 289

Rates of change 289
Infinitesimal changes 291
Successive differentiation 294
Differentiating a complete expression 295
Successive differentiation of movement 297
Circular measure of angles 298
Differential of angles 300
Successive differentiation of sine wave 301
Finding series for sine 302
Finding series for cosine 304
Questions and problems 305

18. Putting differentiation to work 308

Differential of sine waves 308
Sinusoidal motion 309
Harmonic motion 310
Linear or nonlinear relationship 311
Nonlinear relationships 312
Analysis of nonlinear relationships 313
Symmetrical nonlinearity 314
Multiple components of power sinusoids 316
Fourth power term in transfer characteristic 317
Combination of power terms 318
Multiples and powers 319
Formulating expressions to specific requirements 321
Combining algebra and trigonometry 322
Questions and problems 323

19. Developing calculus theory 326

The concept of functions 326
Two functions multiplied together 327

Checking the formula	328
Using the product formula	329
One function divided by another	329
Checking quotient functions	330
Using the quotient formula	330
Function of a function derivative	332
Equation of a circle	333
Successive derivatives of tangent function	334
Integration is the reverse of differentiation	335
Patterns in calculations	336
The constant of integration	336
Definite integrals	337
Finding area by integration	338
Area of a circle	339
Curved areas of cylinders and cones	340
Surface area of sphere	341
Finding volume by integration	341
Volume of a pyramid	341
Volume of cone	343
Volume of sphere	344
<i>Questions and problems</i>	344

20. Combining calculus with other tools 347

Maxima and minima	347
Maximum and minimum points	348
Point of inflection	349
Second derivative gives more information	350
More help from second derivatives	351
Maximum area with constant perimeter	352
Boxes with minimum surface area	352
Cylindrical container with minimum surface area	353
Conical container	355
Equations for circles, ellipses, and parabolas	355
Directrix, focus, and eccentricity	356
The ellipse and the circle	357
Relationships between focus, directrix, and eccentricity	358
Focus property of parabolas	359
Focus property of ellipses	360
Reflection properties of ellipses and parabolas	360
Hyperbolas: eccentricity greater than unity	361
Asymptotes	362
Second-order curves	363
Conic sections produce second-order curves	364
<i>Questions and problems</i>	365

21. Introduction to coordinate systems 366

- Two-dimensional systems of coordinates 366
- Equation of a straight line 367
- Equation for a circle 367
- Three-dimensional systems of coordinates 369
- Equations of line and plane in rectangular coordinates 370
- Equations in spherical polar coordinates 371
- Three-dimensional second-order curves 372
- Questions and problems* 372

Part 4. Developing algebra, geometry, trigonometry, and calculus as analytical methods in mathematics

22. Complex quantities 377

- Imaginary quantities 377
- The complex plane 379
- Complex quantities 379
- Multiplying complex quantities 382
- Reciprocal of complex quantities 382
- Division of complex quantities 383
- Rationalization 384
- Checking results and summarizing 385
- Use of a complex plane 386
- Quadratic roots with complex quantities 388
- Roots by complex quantities 389
- Questions and problems* 389

23. Making series do what you want 392

- A pattern to a series 392
- Pursuing the pattern 393
- Natural growth and decay functions 394
- Value of e 395
- Series for $\arctan x$ 396
- Concept of logarithms 396
- A gap in the series of derivatives 397
- Logarithmic function in calculus 398
- Functions of ϵ 399
- Relationship between exponential and trigonometric series 399
- Convergence of exponential and trigonometric series 400
- Significance of exponential series 400
- Significance of e^{ix} 401
- Complex exponential functions 401
- Complex p plane 403

Complex frequency plane 404
 Hyperbolic functions 404
Questions and problems 406

24. The world of logarithms 409

Logarithmic series 409
 Logarithmic series: modified 410
 Calculating logarithms 412
 Common logarithms 414
 Using logarithms: multiplication and division 415
 Using logarithms: indices 416
 Using logarithms with a formula 418
 Finding the law by logarithms 419
Questions and problems 420

25. Mastering the tricks 422

Trigonometrical series: $\tan x$ 422
 Series for $\sec x$ 423
 Series for $\arcsin x$, $\arccos x$ 423
 Convergence of a series 424
 A useful conversion 425
 Power/multiple conversions 426
 Checking the result 428
 Integration tools: partial fractions 428
 More partial fractions 429
 Product formula in integration 429
 More product formula 431
 Another one by product formula 431
 Changing the variable 432
 Slope on logarithmic scales 434
 A numerical example of slope on log scales 436
 Making the curve fit parameters 436
 Drawing hints 437
Questions and problems 438

26. Development of calculator aids 441

The slide rule 441
 The simple nomogram 442
 Multi-formula nomograms 442
 The ratio nomogram 443
 The reciprocal nomogram 444
 The graphical chart 444
 Change of scales in the graphical chart 446
 Resolving complex quantities graphically 447

Construction for the complex resolution graph 447
Modified linear representation 448
Other possibilities 448
Another concept in chart design 450
Duality between types of calculators 451
Waveform synthesis 451
Fourier series 453
A triangular waveform 453
A square wave 454
Relationship between square and triangular 454
An offset square wave 456
The square wave as a “switching” function 456
Series for quadratic curve 459
The finite approach to the infinite 461
Questions and problems 463

27. Digital mathematics 465

Numbering 466
Decimal system 466
Binary system 467
Octal system 467
Hexadecimal system 467
Logic and Boolean algebra 467
Trinary logic 468
Fuzzy logic 468
Electronic logic gates 468
Basic gates 468
Composite gates 470
Binary circuits and symbols 471
Bits and bytes 471
Flip-flops 472
Compression 473
RGB color model 474
Working with truth tables 475
Building up 475
Breaking down 476
Questions and problems 477

Appendix. Answers to questions and problems 479

Index 559

About the Authors 569

Introduction

This book is intended as a “refresher” course in mathematics for scientists, engineers, and technicians. It begins with a review of arithmetic, and progresses through intermediate and advanced topics, including algebra, trigonometry, geometry, coordinate systems, calculus, differential equations, complex numbers, series, logarithms, and digital logic.

It is assumed that you have already been exposed to the topics in this book. If you haven't — for example, if you have never before seen calculus — you should take a basic course on that subject first, and use this book as a supplement and as a future reference. But maybe you took calculus in college, and that was 20 years ago! The concepts are still in your mind, but they're no longer right up in front. In that case, this book can bring things back to the surface, so you can again work easily with concepts you learned a long time ago.

Each chapter ends with a “Questions and problems” section. You should feel free to refer to the text when solving these problems. Answers are in the appendix. In some cases, descriptions of the problem-solving processes are given in the answer key. Keep in mind that many problems in mathematics can be solved in more than one way. So if you get the right answer by a method that differs from the scheme in the answer key, don't worry. You might even find a better way!

In recent years, electronic calculators have become available to an extent that renders much of the material in this book purely theoretical and “academic”. Computers can render three-dimensional geometric problems to a high degree of accuracy, while providing beautiful color illustrations that you can orient any way you want. To find the sine of an angle or the logarithm of a number, you can punch it up on a calculator you bought for \$6.95 at the department store, and get an answer accurate to 10 decimal places. Nevertheless, it's helpful to understand the theory involved, so you should at least glance at all the material in this book.

Most people are “strong” in certain areas of mathematics, and “weak” in others. In your job, you probably need knowledge of some fields far more than others. If you’re lucky, your strongest knowledge will correspond to the field you use or need the most. But if you’re like most people, there will be differences. For example, I’m pretty good at calculus and analysis, and not so good at probability and statistics. But in my current work, I need to have a functional knowledge of statistics more than I need to differentiate or integrate functions. As a result, I found myself working harder, as I revised this book, on the probability and statistics sections than on the calculus sections. When you use this book as a “refresher” course, keep in mind that you might need intensive work on subjects you don’t like or are not good at.

The material here is presented in a “fast-and-furious” format. There’s a lot of information in a small space. You’ll sometimes find your progress must be measured in hours per page, rather than pages per hour. If you get stuck someplace, don’t worry. Just skip ahead or go back, work on something else for a while, and then come back to the hard stuff. And of course, you can always refer to more basic texts to reinforce your knowledge of subjects where you are weak.

Stan Gibilisco

Acknowledgments

My thanks are extended to Darrel VanderZee, computer consultant and mathematician, for reviewing the manuscript, offering suggestions, and making corrections.

This page intentionally left blank.

Part 1

Arithmetic as an outgrowth of learning to count

This page intentionally left blank.