# A First Course in Stochastic Models

Henk C. Tijms

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# Contents

# Preface

The teaching of applied probability needs a fresh approach. The field of applied probability has changed profoundly in the past twenty years and yet the textbooks in use today do not fully reflect the changes. The development of computational methods has greatly contributed to a better understanding of the theory. It is my conviction that theory is better understood when the algorithms that solve the problems the theory addresses are presented at the same time. This textbook tries to recognize what the computer can do without letting the theory be dominated by the computational tools. In some ways, the book is a successor of my earlier book *Stochastic Modeling and Analysis*. However, the set-up of the present text is completely different. The theory has a more central place and provides a framework in which the applications fit. Without a solid basis in theory, no applications can be solved. The book is intended as a first introduction to stochastic models for senior undergraduate students in computer science, engineering, statistics and operations research, among others. Readers of this book are assumed to be familiar with the elementary theory of probability.

I am grateful to my academic colleagues Richard Boucherie, Avi Mandelbaum, Rein Nobel and Rien van Veldhuizen for their helpful comments, and to my students Gaya Branderhorst, Ton Dieker, Borus Jungbacker and Sanne Zwart for their detailed checking of substantial sections of the manuscript. Julian Rampelmann and Gloria Wirz-Wagenaar were helpful in transcribing my handwritten notes into a nice Latex manuscript.

Finally, users of the book can find supporting educational software for Markov chains and queues on my website http://staff.feweb.vu.nl/tijms.

CHAPTER 1

# The Poisson Process and Related Processes

## 1.0 INTRODUCTION

The Poisson process is a counting process that counts the number of occurrences of some specific event through time. Examples include the arrivals of customers at a counter, the occurrences of earthquakes in a certain region, the occurrences of breakdowns in an electricity generator, etc. The Poisson process is a natural modelling tool in numerous applied probability problems. It not only models many real-world phenomena, but the process allows for tractable mathematical analysis as well.

The Poisson process is discussed in detail in Section 1.1. Basic properties are derived including the characteristic memoryless property. Illustrative examples are given to show the usefulness of the model. The compound Poisson process is dealt with in Section 1.2. In a Poisson arrival process customers arrive singly, while in a compound Poisson arrival process customers arrive in batches. Another generalization of the Poisson process is the non-stationary Poisson process that is discussed in Section 1.3. The Poisson process assumes that the intensity at which events occur is time-independent. This assumption is dropped in the non-stationary Poisson process. The final Section 1.4 discusses the Markov modulated arrival process in which the intensity at which Poisson arrivals occur is subject to a random environment.

## 1.1 THE POISSON PROCESS

There are several equivalent definitions of the Poisson process. Our starting point is a sequence $X_1, X_2, \ldots$ of positive, independent random variables with a common probability distribution. Think of $X_n$ as the time elapsed between the $(n-1)$th and $n$th occurrence of some specific event in a probabilistic situation. Let

$$S_0 = 0 \quad \text{and} \quad S_n = \sum_{k=1}^{n} X_k, \quad n = 1, 2, \ldots .$$

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