Theory and Applications of Stochastic Differential Equations

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Contents

CHAPTER 1

Review of Probability Theory

1.1 EVENTS AND SAMPLE SPACES

Consider the experiment of tossing a fair coin three times. The possible outcomes of this experiment are HHH, HHT, HTH, THH, HTT, THT, TTH, and TTT, where H denotes heads and T denotes tails. Each possible outcome of the experiment is called an *elementary event*. Thus there are eight elementary events in the experiment of tossing a coin three times. More complicated events can be expressed as combinations of the elementary events. Thus the event "two or more heads turn up" in the coin-tossing experiment, which we denote by D, consists of the elementary events HHH, HHT, HTH, THH; that is,

$$D = \{\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}\}.$$

The set Ω of all elementary events corresponding to an experiment is called a *sample space* and each elementary event, denoted by ω, is called a *point* in Ω. In the particular example under consideration, we have

$$\Omega = \{\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \text{TTH}, \text{THT}, \text{HTT}, \text{TTT}\}.$$

We write $\omega \in \Omega$ to read "ω is a point in (or an element of) Ω." Any event A that consists of elementary events is a subset of Ω. In particular, the impossible event $\varnothing$, that is, an event that contains no elementary events, is called *the empty* set. Let A and B be events in Ω; then we say that *A is a subset of B* if every element of A is also an element of B, and we write $A \subset B$. Obviously, $A \subset \Omega$, $\varnothing \subset A$, and $A \subset A$. For example, the set D in the coin-tossing experiment is a subset of the set (event) E: "at least one head