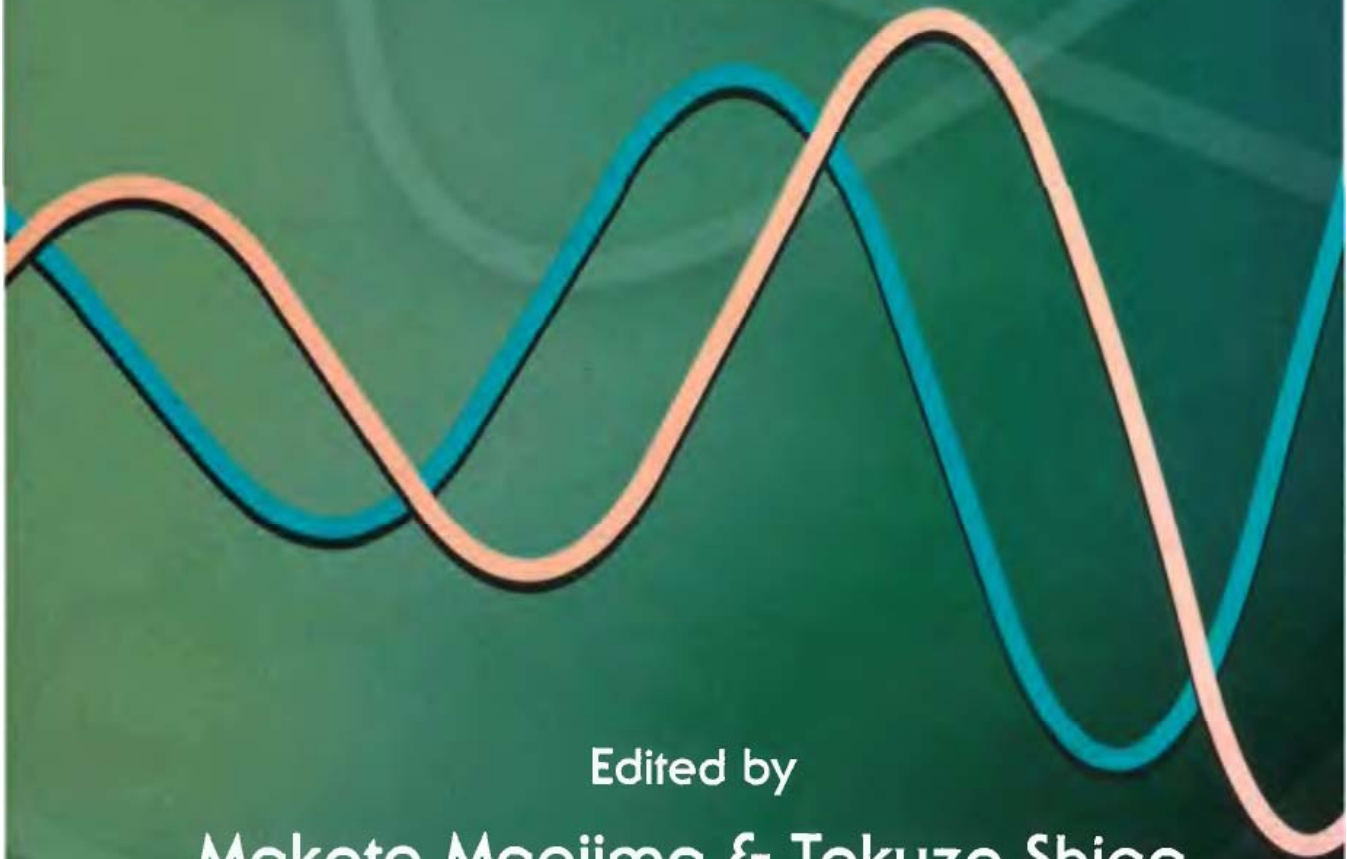


STOCHASTIC PROCESSES

Selected Papers of Hiroshi Tanaka



Edited by

Makoto Maejima & Tokuzo Shiga

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Hiroshi Tanaka

Preface

Hiroshi Tanaka is known for his outstanding contributions to the theory of stochastic processes. Since his first paper on diffusion processes in 1957, he has obtained brilliant mathematical results including construction of diffusion processes with continuous coefficients, Tanaka formula for Brownian motion, Skorohod equation on a convex domain, Boltzmann equations, diffusion processes in random media and so on.

On the occasion of his retirement from academic positions, we planned to collect his important contributions to the theory of stochastic processes in one volume, making his works available in a unified form to the mathematical community.

Besides his selected papers in this volume we asked Professors Henry McKean, Marc Yor, Shinzo Watanabe and Tanaka himself to write some essays on Tanaka's mathematics and personality.

Among Tanaka's works the most popular one would be the celebrated Tanaka formula for Brownian local time, which was never published. The story around the birth of Tanaka formula can be found in the essays of Marc Yor and Shinzo Watanabe.

As editors of this volume, we are grateful to World Scientific Publishing Co. Pte. Ltd. for the production of the volume. Our special thanks go to Professors Henry McKean, Marc Yor and Shinzo Watanabe for their writing interesting essays, to Professors Nobuyuki Ikeda and Yuji Ito for their valuable suggestions in the process of planning this volume, to Professors Yozo Tamura, Hideki Tanemura and Yuki Suzuki for their assistance throughout its preparation.

Makoto Maejima
Tokuzo Shiga

Contents

Preface	vii
H. Tanaka: An Appreciation (by Henry McKean)	1
From Local Times to Random Environments ... (by Marc Yor)	2
Contributions and Influences of Professor Tanaka in Stochastic Analysis (by Shinzo Watanabe)	5
Some Comments on My Mathematical Works in Retrospect (by Hiroshi Tanaka)	10
[5] [†] Additive Functionals of the Brownian Path (with H. P. McKean)	19
[7] Note on Continuous Additive Functionals of the 1-Dimensional Brownian Path	47
[8] Existence of Diffusions with Continuous Coefficients	54
[14] Propagation of Chaos for Certain Purely Discontinuous Markov Processes with Interactions	69
[15] An Inequality for a Functional of Probability Distributions and Its Application to Kac's One-Dimensional Model of a Maxwellian Gas	83
[16] On Markov Process Corresponding to Boltzmann's Equation of Maxwellian Gas	89
[19] On the Uniqueness of Markov Process Associated with the Boltzmann Equation of Maxwellian Molecules	101
[22] Probabilistic Treatment of the Boltzmann Equation of Maxwellian Molecules	118

[†]Numbers in brackets refer to the Bibliography of Hiroshi Tanaka (see pages 425–428)

[24] Stochastic Differential Equations with Reflecting Boundary Condition in Convex Regions	157
[27] Some Probabilistic Problems in the Spatially Homogeneous Boltzmann Equation	172
[29] Limit Theorems for Certain Diffusion Processes with Interaction . . .	182
[32] Central Limit Theorem for a System of Markovian Particles with Mean Field Interactions (with T. Shiga)	202
[33] Propagation of Chaos for Diffusing Particles of Two Types with Singular Mean Field Interaction (with M. Nagasawa)	223
[34] Stochastic Differential Equations for Mutually Reflecting Brownian Balls (with Y. Saisho)	238
[35] Limit Distribution for 1-Dimensional Diffusion in a Reflected Brownian Medium	254
[36] Limit Distributions for One-Dimensional Diffusion Processes in Self-Similar Random Environments	270
[38] Stochastic Differential Equation Corresponding to the Spatially Homogeneous Boltzmann Equation of Maxwellian and Non-Cutoff Type	292
[41] Limit Theorem for One-Dimensional Diffusion Process in Brownian Environment	311
[49] On the Maximum of a Diffusion Process in a Drifted Brownian Environment (with K. Kawazu)	328
[51] Recurrence of a Diffusion Process in a Multidimensional Brownian Environment	336
[52] Localization of a Diffusion Process in a One-Dimensional Brownian Environment	341
[54] Diffusion Processes in Random Environments	353

[55] Environment-Wise Central Limit Theorem for a Diffusion in a Brownian Environment with Large Drift	361
[56] A Diffusion Process in a Brownian Environment with Drift (with K. Kawazu)	373
[57] Limit Theorems for a Brownian Motion with Drift in a White Noise Environment	396
[58] Invariance Principle for a Brownian Motion with Large Drift in a White Noise Environment (with K. Kawazu)	406
[63] Some Theorems Concerning Extrema of Brownian Motion with d -Dimensional Time	415
Bibliography of Hiroshi Tanaka	425
Permissions	429

H. Tanaka: An Appreciation

by Henry McKean

It is a pleasant thing for me to write this little note to accompany the selected papers of H. Tanaka collected in this volume.

When I was young and fresh out of school, I visited Kyôto for a year (1957/58) at the invitation of K. Itô. My great-grandfather (whom I never knew) was an amateur of Japanese art, generally, and of Okyô in particular. Part of his collection came down to my family, so even when I was quite little, I was fascinated by Japan and things Japanese. The double chance to see Japan and to work with Itô was an irresistible piece of luck and I was pretty well prepared for that.

What I was not prepared for, but happily surprised by, was the enthusiastic crowd of young Japanese probabilists under Itô's wing who came faithfully, once a week, to hear the new developments in Brownian motion and diffusion stemming from the ideas of W. Feller and E.B. Dynkin — “sowing the seeds of diffusion in the mathematical fields of Japan” as Itô put it. H. Tanaka stood out in this company for the quickness of his understanding, for his mathematical taste, and for his quiet but strong personality. That was our first acquaintance. Later he visited me at MIT in Cambridge; in between, we collaborated on additive functions of the Brownian path ([5] †, 1961).

Tanaka's mathematical production is marked by the elegant use of common ideas employed in novel ways and with uncommon skill. No better example can be found than his 1963 paper ([7]) in which Itô's lemma is used with a bit of extra audacity to prove a wonderful formula, simple but highly effective, for the Brownian local time. I would like particularly to cite the deep series of papers ([15], [16], [19], [27]) on Boltzmann's equation for the Maxwellian gas and the Markov process that underlies it, a subject dear to my heart. This had been initiated by Kaç and studied somewhat by me, but it was Tanaka who really understood it correctly, and everything of Tanaka's shows like elegance, simplicity, depth.

This note is to express to Tanaka my admiration and friendship and to wish him health and happy productive years still to come.

†Numbers in brackets refer to the Bibliography of Hiroshi Tanaka (see pages 425–428)