



## Preface

The purpose of this handbook is to supply a collection of mathematical formulas and tables which will prove to be valuable to students and research workers in the fields of mathematics, physics, engineering and other sciences. To accomplish this, care has been taken to include those formulas and tables which are most likely to be needed in practice rather than highly specialized results which are rarely used. Every effort has been made to present results concisely as well as precisely so that they may be referred to with a maximum of ease as well as confidence.

Topics covered range from elementary to advanced. Elementary topics include those from algebra, geometry, trigonometry, analytic geometry and calculus. Advanced topics include those from differential equations, vector analysis, Fourier series, gamma and beta functions, Bessel and Legendre functions, Fourier and Laplace transforms, elliptic functions and various other special functions of importance. This wide coverage of topics has been adopted so as to provide within a single volume most of the important mathematical results needed by the student or research worker regardless of his particular field of interest or level of attainment.

The book is divided into two main parts. Part I presents mathematical formulas together with other material, such as definitions, theorems, graphs, diagrams, etc., essential for proper understanding and application of the formulas. Included in this first part are extensive tables of integrals and Laplace transforms which should be extremely useful to the student and research worker. Part II presents numerical tables such as the values of elementary functions (trigonometric, logarithmic, exponential, hyperbolic, etc.) as well as advanced functions (Bessel, Legendre, elliptic, etc.). In order to eliminate confusion, especially to the beginner in mathematics, the numerical tables for each function are separated. Thus, for example, the sine and cosine functions for angles in degrees and minutes are given in separate tables rather than in one table so that there is no need to be concerned about the possibility of error due to looking in the wrong column or row.

I wish to thank the various authors and publishers who gave me permission to adapt data from their books for use in several tables of this handbook. Appropriate references to such sources are given next to the corresponding tables. In particular I am indebted to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Oliver and Boyd Ltd., Edinburgh, for permission to use data from Table III of their book *Statistical Tables for Biological, Agricultural and Medical Research*.

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M. R. SPIEGEL

Rensselaer Polytechnic Institute  
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*Part I*

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