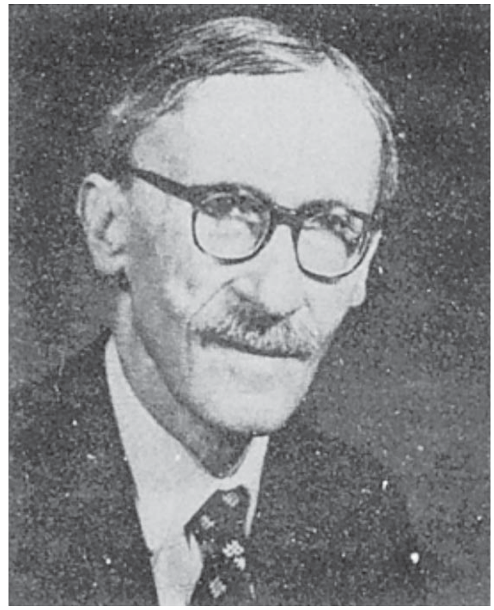
Doob

Polya

Kolmogorov

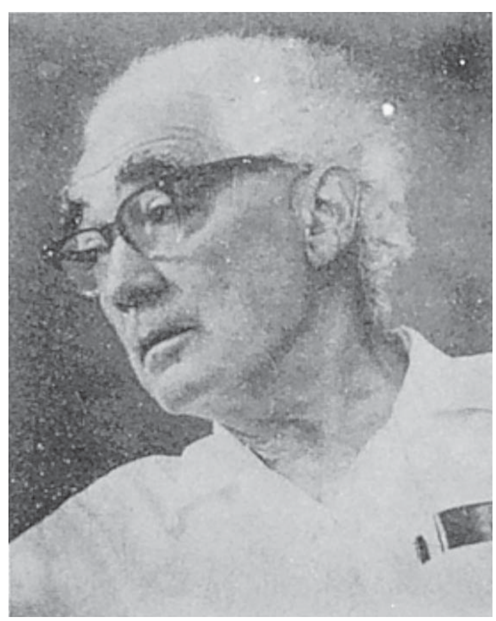
Cramer

Borel



Levy

Keynes



Feller

Contents

Preface to the Fourth Edition

In this edition two new chapters, 9 and 10, on mathematical finance are added. They are written by Dr. Farid AitSahlia, *ancien élève*, who has taught such a course and worked on the research staff of several industrial and financial institutions.

The new text begins with a meticulous account of the uncommon vocabulary and syntax of the financial world; its manifold options and actions, with consequent expectations and variations, in the marketplace. These are then expounded in clear, precise mathematical terms and treated by the methods of probability developed in the earlier chapters. Numerous graded and motivated examples and exercises are supplied to illustrate the applicability of the fundamental concepts and techniques to concrete financial problems. For the reader whose main interest is in finance, only a portion of the first eight chapters is a "prerequisite" for the study of the last two chapters. Further specific references may be scanned from the topics listed in the Index, then pursued in more detail.

I have taken this opportunity to fill a gap in Section 8.1 and to expand Appendix 3 to include a useful proposition on martingale stopped at an optional time. The latter notion plays a basic role in more advanced financial and other disciplines. However, the level of our compendium remains *elementary*, as befitting the title and scheme of this textbook. We have also included some up-to-date financial episodes to enliven, for the beginners, the stratified atmosphere of "strictly business". We are indebted to Ruth Williams, who read a draft of the new chapters with valuable suggestions for improvement; to Bernard Bru and Marc Barbut for information on the Pareto-Lévy laws originally designed for income distributions. It is hoped that a readable summary of this renowned work may be found in the new Appendix 4.

Kai Lai Chung
August 3, 2002

Prologue to Introduction to Mathematical Finance

The two new chapters are self-contained introductions to the topics of mean-variance optimization and option pricing theory. The former covers a subject that is sometimes labeled "modern portfolio theory" and that is widely used by money managers employed by large financial institutions. To read this chapter, one only needs an elementary knowledge of probability concepts and a modest familiarity with calculus. Also included is an introductory discussion on stable laws in an applied context, an often neglected topic in elementary probability and finance texts. The latter chapter lays the foundations for option pricing theory, a subject that has fueled the development of finance into an advanced mathematical discipline as attested by the many recently published books on the subject. It is an initiation to martingale pricing theory, the mathematical expression of the so-called "arbitrage pricing theory", in the context of the binomial random walk. Despite its simplicity, this model captures the flavors of many advanced theoretical issues. It is often used in practice as a benchmark for the approximate pricing of complex financial instruments.

I would like to thank Professor Kai Lai Chung for inviting me to write the new material for the fourth edition. I would also like to thank my wife Unnur for her support during this rewarding experience.

Farid AitSahlia
November 1, 2002

1

Set

1.1. Sample sets

These days schoolchildren are taught about sets. A second grader* was asked to name "the set of girls in his class." This can be done by a complete list such as:

"Nancy, Florence, Sally, Judy, Ann, Barbara, . . . "

A problem arises when there are duplicates. To distinguish between two Barbaras one must indicate their family names or call them B_1 and B_2. The same member cannot be counted twice in a set.

The notion of a set is common in all mathematics. For instance, in geometry one talks about "the set of points which are equidistant from a given point." This is called a circle. In algebra one talks about "the set of integers which have no other divisors except 1 and itself." This is called the set of prime numbers. In calculus the domain of definition of a function is a set of numbers, e.g., the interval (a, b); so is the range of a function if you remember what it means.

In probability theory the notion of a set plays a more fundamental role. Furthermore we are interested in very general kinds of sets as well as specific concrete ones. To begin with the latter kind, consider the following examples:

(a) a bushel of apples;
(b) fifty-five cancer patients under a certain medical treatment;

*My son Daniel.