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Analysis of Stochastic Models in Manufacturing Systems Pertaining to Repair Machine Failure

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This chapter deals with three stochastic models *A*, *B*, and *C*, each consisting of two nonidentical units in standby network. One unit is named as the priority unit (*p*-unit) and the other as the nonpriority or ordinary unit (*o*-unit). In each model, the *p*-unit gets priority in operation over the *o*-unit. A single server is available to repair a failed unit and a failed repair machine (R.M.). The R.M. is required to do the repair of a failed unit. In models *A* and *C*, the *o*-unit gets priority in repair over the *p*-unit, whereas in model-*B* the priority in repair is also given to the *p*-unit over the *o*-unit. In each model it is assumed that the R.M. may also fail during its working and then the preference in repair is given to R.M. over any of the units. In models *A* and *B*, the failure and repair times of each unit are assumed to be uncorrelated independent random variables (r.vs.), whereas in model *C* these two r.vs. are assumed to be correlated having bivariate exponential distribution. In each model we have obtained various economic measures of system effectiveness by using the regenerative point technique.

7.1 Introduction

Two-unit standby systems have been widely studied in the literature of reliability due to their frequent and significant use in modern business and industry. Various authors including [1, 3, 8–10, 17–23, 25] have studied two-unit standby systems with different sets of assumptions and obtained various characteristics of interest by using the theories of semi-Markov process, regenerative process, Markov-renewal process and supplementary variable technique. They have given equal priority to both the units in respect of operation and repair. But realistic situations may arise when it is necessary to give priority to the main unit in respect of operation and repair as compared to the ordinary (standby) unit. A very good example of this situation is that of a system consisting of two units, one power supply and the other generator. The priority is obviously given to the power through power station rather than generator. The generator will be used only when the power supply through power station is discontinued. Further, due to costly operation of the generator, the priority in repair may be given to power station rather than the generator.

Keeping the above concept in view, Nakagawa and Osaki [24] have studied the behavior of a two-unit (priority and ordinary) standby system with two modes of each unit — normal and total failure. Goel et al. [2] have obtained the cost function in respect of a two-unit priority standby system with imperfect switching device. They have assumed general distributions of failure and repair times of each unit. Recently, Gupta and Goel [11] investigated a two-unit priority standby system model under the assumption that whenever an operative unit fails, a delay occurs in locating the repairman and having him available to repair a failed unit/system. Some other authors including [12–15] have also investigated two-unit priority standby system models under different sets of assumptions. The common assumption in all the above models is that a single repairman is considered and the preference with respect to operation and repair is given to priority (*p*) unit over the ordinary (*o*) unit. However, situations may also arise when one is to provide preference to priority (*p*) unit only in operation and not in repair. Regarding the repair, either the preference may be given to *o*-unit over the *p*-unit or the repair discipline may be first come first serve (FCFS). So, more recently Gupta et al. [16] investigated a two nonidentical unit cold standby system model assuming that the preference in operation is given to the first unit (*p*-unit) while in repair the preference is given to the second unit (*o*-unit). The system model under this study can be visualised by a very simple example: Suppose in a two-unit cold standby system model two nonidentical units are an air conditioner (A.C.) and an air cooler. Obviously the preference in operation will be given to the A.C. and air cooler will get the preference in repair as the repair of A.C. is costly and time-consuming. The case of standby redundant system is not seen in the literature of reliability when the preference in operation is given to *p*-unit but in repair the policy is FCFS.

All the above discussed authors have analysed the system models under the assumptions that the machine/device used for repairing a failed unit remains good forever. In real situations this assumption is not always practicable as the repair machine (R.M.) may also have a specified reliability and can fail during the repair process of a failed unit. For example, in the case of nuclear reactors, marine equipments, etc., the robots are used for the repair of such type of systems. It is evident that a robot, a machine, may fail while performing its intended task. In this case obviously the repairman first repairs the repair machine and then takes up the failed unit for repair.

In this chapter we discuss three system models, *A*, *B*, and *C*, each consisting of two nonidentical units named as *p*-unit and *o*-unit. It is assumed that in each model the *p*-unit gets priority in operation as only one unit is sufficient to do the required job. A repair machine (R.M.) is required to do the repair of a failed unit which can also fail during its operation. Further, a single repairman is available to repair a failed unit as well as a failed R.M. and in each model the priority is given to R.M. over any of the failed units. Regarding the repair of failed units, it is assumed in model *B* that the *p*-unit gets preference in repair over the *o*-unit, whereas in models *A* and *C* the priority in repair is given to the *o*-unit rather than to the *p*-unit. In models *A* and *B*, the basic assumption is that the failure and repair times are taken uncorrelated independent r.vs. However, a common experience of system engineers and managers reveals that in many system models there exists some sort of correlation between failure and repair times. It is observed that in most of the system models an early (late) failure leads to early (delayed) repair. The concept of linear relationship is the main point of consideration. Therefore, taking this concept in view, in model *C*, the joint distribution of failure and repair times is assumed to be bivariate exponential (B.V.E) of the form suggested by "Paulson" ($0 \leq r \leq 1$). The p.d.f. of the B.V.E. is

$$f(x, y) = \alpha\beta(1 - r)e^{-\alpha x - \beta y} I_0(2\sqrt{\alpha\beta r \times y}) \qquad (1)$$
$$x, y, \alpha, \beta > 0, \quad 0 \leq r < 1$$

where $I_0(z) = \Sigma_{k=0}^{\infty} \frac{(Z/2)^{2K}}{(K!)^2}$ is the modified Bessel function of type I and order Zero. Some authors including [4–7,16] have already analysed system models by using the above mentioned concept.

Using regenerative point technique in the Markov renewal process, the following reliability characteristics of interest to system designers and operation managers have been obtained for models *A*, *B*, and *C*.

(i) reliability of the system and mean time to system failure (MTSF);
(ii) pointwise and steady state availabilities of the system;
(iii) the probability that the repairman is busy at an epoch and in steady state;
(iv) expected number of repairs by the repairman in $(0, t)$ and in steady state; and
(v) expected profit incurred by the system in $(0, t)$ and in steady state.

Some of the above characteristics have also been studied and compared through graphs and important conclusions have been drawn in order to select the most suitable model under the given conditions.

7.2 System Description and Assumptions

(i) The system is comprised of two nonidentical units and a repair machine (R.M.). The units are named as priority (*p*) unit and ordinary (*o*) unit. The operation of only one unit is sufficient to do the job.
(ii) In each model the *p*-unit gets priority in operation over the *o*-unit. The *o*-unit operates only when *p*-unit has failed. So, initially the p-unit is operative and *o*-unit is kept as cold standby which cannot fail during its standby state.
(iii) Each unit of the system has two modes normal (*N*) and total failure (*F*). A switching device is used to put the standby unit into operation and its functioning is always perfect and instantaneous.
(iv) A single repairman is available with the system to repair a failed unit and failed R.M. In models *A* and *C*, the *o*-unit gets priority in repair over the *p*-unit, whereas in model *B*, the priority in repair is given to the *p*-unit over the *o*-unit. Further, the R.M. gets the preference in repair over both the units.
(v) The R.M. repairs a failed unit and it can also fail during the repair of a unit. In such a situation the repair of the failed unit is discontinued and the repairman starts the repair of the R.M. as a single repairman is available. Each repaired unit and R.M. work as good as new.
(vi) The R.M. is good initially and it cannot fail until it begins functioning.
(vii) In models *A* and *B*, the failure times and repair times of a unit and R.M. are assumed to be independent and uncorrelated r.vs., whereas in model *C* the failure and repair times of the units are correlated r.vs.

(viii) In models A and B, the failure time distributions of each unit and the R.M. are taken to be negative exponential with different parameters while all the repair time distributions are general having different probability density function (p.d.f). Further, in these models it is assumed that the restarted repair (after interruption) of a unit is preemptive repeat type having the same p.d.f as that of the fresh repair.

(ix) In model C, the failure and repair times of each unit are jointly distributed having the bivariate exponential density function of the form (1.1) with different parameters. Due to the priority in repair, the repairs of the p-unit and the o-unit are interrupted many times. When such a unit is again taken up for repair, then the time required to complete the repair now is known as residual repair time of the unit concerned. The residual repair time of each unit need not depend on its failure time and the random variable denoting it also has a negative exponential distribution.

(x) The failure and repair time distributions of R.M. in model C are taken to be negative exponential with different parameters.

7.3 Notation and States of the System

E_0 : initial state of the system i.e., the state at time $t = 0$

E : set of regenerative states

$\bar{E}$: complementary set of E

$q_{ij}(.), Q_{ij}(.)$: p.d.f. and c.d.f. of one step or direct transition time from state $S_i \in E$ to $S_j \in$ E.

p_{ij} : [1]steady-state transition probability from state S_i to S_j such that $p_{ij} = Q_{ij}(\propto) = \int q_{ij}(u)\, du$

$q_{ij}^{(k,l)}(.), Q_{ij}^{(k,l)}(.)$: p.d.f. and c.d.f. of transition time from state $S_i \in E$ to $S_j \in E$ via states $S_k \in \bar{E}$ and $S_l \in \bar{E}$

$p_{ij}^{(k,l)}$: steady-state transition probability from state $S_i \in E$ to $S_j \in E$ via states $S_k \in \bar{E}$ and $S_l \in \bar{E}$ such that

$$p_{ij}^{(k,l)} = Q_{ij}^{(k,l)}(\infty) = \int q_{ij}^{(k,l)}(u)\,du$$

$Q_{ij|x}(.)$: c.d.f. of transition time from state $S_i \in \bar{E}$ to $S_j \in E$ given that the unit under repair in state S_i entered into F-mode after an operation of time x (for model-C)

$p_{ij|x}$: steady-state probability of transition from state $S_i \in \bar{E}$ to $S_j \in E$ given that the unit under repair in state S_i entered into F-mode after an operation of time x (for model-C)

$$= \lim_{t \to \infty} Q_{ij|x}(t) = \lim_{s \to 0} \tilde{Q}_{ij|x}(s)$$

$Z_i(t)$: probability that the system sojourns in state S_i up to time t.

ψ_i : mean sojourn time in state S_i

$$= \int Z_i(t)\,dt = \lim_{s \to 0} Z_i^*(s)$$

$\psi_{i|x}$: mean sojourn time in state $S_i \in \bar{E}$ given that the unit under repair in this state entered into F-mode after an operation of time x (for model-C)

$*, \sim$: symbols for Laplace and Laplace Stieltjes Transforms (LT and LST).

[1]The limits of the integration are not mentioned throughout the chapter whenever they are 0 to ∞.

©, Ⓢ : symbols for ordinary and Stieltjes convolutions

i.e., $$A(t)©B(t) = \int_0^t A(u)B(t-u)du$$

$$A(t)ⓈB(t) = \int_0^t dA(u)B(t-u)$$

α_1, α_2 : constant failure rates of p and o-unit, respectively (for models A&B)
$G_1(.), G_2(.)$: c.d.f. of the repair time of p and o-unit, respectively (for models A&B)
λ : constant failure rate of R.M.
$H(.)$: c.d.f. of time to repair of R.M. (for models A&B)
μ : constant repair rate of R.M. (for model C)
X_1, Y_1 : r.vs. denoting the life time and repair time, respectively, of p-unit (for model C)
X_2, Y_2 : r.vs. denoting the life time and repair time, respectively, of o-unit (for model C)
Y_1', Y_2' : r.vs. denoting the residual repair times of p and 0-unit, respectively each having the negative exponential distribution with parameters β_1 and β_2 (for model C)
$f_i(x, y)$: joint p.d.f. of (X_i, Y_i)
$i = 1, 2$ $$= \alpha_i\beta_i(1-r_i)e^{-(\alpha_i x+\beta_i y)}I_0(2\sqrt{\alpha_i\beta_i r_i xy})$$

so that the conditional p.d.f. of Y_i given $X_i = x$ is

$$\beta_i e^{-(\beta_i y+\alpha_i r_i x)}I_0(2\sqrt{\alpha_i\beta_i r_i xy})$$

where

$$I_0(2\sqrt{\alpha_i\beta_i r_i xy}) = \sum_{j=0}^{\infty}\frac{(\alpha_i\beta_i r_i xy)^j}{(j!)^2}$$

$$x \geq 0, y \geq 0, \quad 0 \leq r_i \leq 1$$

$q_i(x)$: marginal p.d.f. of X_i

$$= \alpha_i(1-r_i)e^{-\alpha_i(1-r_i)x}$$

Symbols used to represent the states of the system:

N_{10}, N_{20} : p and o-unit in N-mode and operative
N_{2s} : o-unit in N-mode and standby
F_{1r}, F_{2r} : p and o-unit in F-mode and under repair
F_{1w}, F_{2w} : p and o-unit in F-mode and waiting for repair
RM_g, PM_o, RM_r : R.M. in good condition, operative, and under repair
$F_{1r'}, F_{2r'}$: p and o-unit in F-mode and again taken up for repair after interruption
$F_{1w'}, F_{2w'}$: p and o-unit in F-mode and waiting for repair after interruption

Considering the above symbols for the two units and the assumptions stated earlier, we have the following states of system models A, B, and C.