

EE3561 - Tutorials 7

①

Runge-kutta Method to
Solve ODE

2nd-order Runge-kutta method (RK2) :-

$$k_1 = h f(x, y)$$

$$k_2 = h f(x+h, y+k_1)$$

$$y(x+h) = y(x) + \frac{1}{2}(k_1 + k_2)$$

4th-order Runge-kutta method (RK4) :-

$$k_1 = h f(x, y)$$

$$k_2 = h f\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right)$$

$$k_3 = h f\left(x + \frac{h}{2}, y + \frac{k_2}{2}\right)$$

$$k_4 = h f\left(x + h, y + k_3\right)$$

$$y(x+h) = y(x) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Q1: Use Rk2 to solve

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$$\frac{dy}{dx} = yx^2 - 1.1y \quad \text{over the interval from } x=0 \text{ to } 1$$

where $y(0)=1$ and $h=0.5$.

Solution step 1, $i=0$

$$k_1 = h f(x_0, y_0) = 0.5 [y_0 x_0^2 - 1.1 y_0] = 0.5 [1(0)^2 - 1.1(1)]$$

$$k_1 = -0.55$$

x	y
$x_0 = 0$	$y_0 = 1$
$x_1 = x_0 + h = 0.5$	$y_1 = y(0.5) = 0.629375$
$x_2 = x_1 + h = 1$	$y_2 = y(1) = 0.4866$

$$k_2 = h f(x_0 + h, y_0 + k_1) = 0.5 f(0.5, 0.45)$$

$$= 0.5 [0.45(0.5)^2 - 1.1(0.45)] = -0.19125$$

$$y(x_0 + h) = y(0.5) = y(0) + \frac{1}{2} (k_1 + k_2)$$

$$y(0.5) = 1 + \frac{1}{2} (-0.55 - 0.19125) = 0.629375$$

step 2 $\therefore i=1$

$$k_1 = h f(x_1, y_1) \quad , \quad x_1 = x_0 + h = 0.5$$

$$k_1 = 0.5 f(0.5, 0.6293) = 0.5 [0.6293(0.5)^2 - 1.1(0.6293)]$$

$$k_1 = -0.2675$$

$$k_2 = h f(x_1 + h, y_1 + k_1) = 0.5 f(1, (0.6293 - 0.2675))$$

$$= 0.5 f(1, 0.3619) = 0.5 [0.3619(1)^2 - 1.1(0.3619)]$$

$$k_2 = -0.018$$

$$y_2 = y_1 + \frac{1}{2}(k_1 + k_2)$$

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$$= 0.62937 + \frac{1}{2}(-0.2675 - 0.0181)$$

$$\boxed{y_2 = 0.4866}$$

$$\Rightarrow y(1) = 0.4866$$

Q2 :- Use Runge-kutta method (Rk2) to

$$\text{Solve } \frac{dy}{dx} = \sin(x+y) - e^x$$

$$y(0) = 4, h = 0.1. \text{ Find } y(0.2)?$$

Solution

x	y
$x_0 = 0$	$y_0 = 4$
$x_1 = x_0 + h = 0.1$	$y_1 = y(0.1) =$
$x_2 = 0.2$	$y_2 = y(0.2) =$

Step 1 $i=0$

$$k_1 = h f(x_0, y_0) = 0.1 f(0, 4)$$

$$= 0.1 [\sin(0+4) - e^0] = \boxed{-0.1756 = k_1}$$

$$k_2 = h f(x_1, y_0 + k_1) = 0.1 f(0.1, (4 - 0.1756))$$

$$= 0.1 f(0.1, 3.824)$$

$$= 0.1 [\sin(0.1 + 3.824) - e^{0.1}] = \boxed{-0.18 = k_2}$$

$\frac{4}{7}$

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$$

$$= 4 + \frac{1}{2}(-0.1756 - 0.181) = \boxed{3.8216 = y_1}$$

$$y_1 = y(0.1) = \underline{\underline{3.8216}}$$

* step 2: $i=1$

$$k_1 = h f(x_1, y_1) = 0.1 f(0.1, 3.8216)$$

$$\boxed{k_1 = -0.18}$$

$$k_2 = h f(x_2, y_1 + k_1) = 0.1 f(0.2, (3.8216 - 0.18))$$

$$k_2 = 0.1 f(0.2, 3.6408) = \boxed{-0.1865 = k_2}$$

$$y_2 = y_1 + \frac{1}{2}(k_1 + k_2) = 3.8216 + \frac{1}{2}(-0.18 - 0.1865)$$

$$y_2 = 3.638$$

$$\Rightarrow \boxed{y(0.2) = 3.638}$$